

Mark Scheme

Summer 2023

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 31 Statistics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 50.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - B marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

- 4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
 - If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
- 6. Ignore wrong working or incorrect statements following a correct answer.
- 7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
 - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any
 A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side

Qu 1	Scheme	Marks	AO
(a)	$[0.13 + 0.25 =] \underline{0.38}$	B1	1.1b
(b)	Independence implies:	(1)	
(b)	e.g. $\left[P(B \cap C) = P(B) \times P(C) \implies \right] 0.3 = (0.3 + 0.05 + 0.25) \times (0.3 + p)$	M1	1.1b
	[Sum of probabilities = 1 gives] So $p = \underline{0.07}$	A1 B1ft (3)	1.1b 1.1b
(c)	$P(A B') = \frac{P(A \cap B')}{P(B')} \text{ or } \frac{0.13}{(1-0.6) \text{ or } (0.13 + "0.2" + "0.07")}$	M1	1.1b
	$=\frac{13}{\underline{40}} \text{ or } \underline{0.325}$	A1	1.1b
		(2)	
	Notes	(6 m	arks)
(a)	B1 for 0.38 (or exact equivalent)		
	If answers are given on Venn Diagram <u>and</u> in the script then the script	takes pred	cedence.
(b)	M1 for a correct equation in p or $P(C)$ only. May be implied by an answer of $p = 0.2$ provided this does not come from incorrect working. Condone missing brackets if they get 0.2 Other rules for independence will give simple rearrangements of this equation.		
Beware	If $p = 0.2$ comes from incorrect working, we've seen $p = \frac{0.6}{0.3} = 0.2$, so	ore M0A0	
	A1 for $p = 0.2$ (or exact equivalent) B1ft for $q = 0.07$ (or exact equivalent) ft their p i.e. $q = 0.27 - 0.2$ " when	re 0 " p "	0.27
(c)	 M1 for a correct ratio of probability expressions or a correct ratio of probabilities ft their values of p and q (provided both probabilities) or letters p and q A1 for 0.325 or exact equivalent. Correct answer only will score 2/2 NB on epen this is labelled M1 but treat it as A1 		

Qu 2	Scheme	Ma	rks	AO
(a)	Comment in context about either independence or random packing e.g. "prizes must be placed in packets at random/independently of each other" or about constant probability e.g.	B1		3.5b
(b)(i)	"the <u>probability</u> of a <u>packet</u> containing a <u>prize</u> is <u>constant/ the same/fixed</u> " $[P(T=6) =] 0.17273 \text{ awrt } \underline{0.173}$	B1	(1)	1.1b
(6)(1)	[1(1 0)] 0.17275 awit <u>0.175</u>	D1		1.10
(ii)	$[P(T < 3) = P(T_{*}, 2) =] 0.061587$ awrt <u>0.0616</u>	B1	(2)	1.1b
(c)	[K = no. of boxes with fewer than 3 packets containing a prize] $K \sim B(5, \text{``}0.0616\text{''})$	M1		1.1b
	P(K = 2) = 0.031344 in the range [0.0313~0.0314]	A1	(2)	1.1b
(d)	$\mathbf{H}_0: p = \frac{1}{7} \mathbf{H}_1: p < \frac{1}{7}$	B1		2.5
	[X = no of packets containing a prize] $X \sim B(110, \frac{1}{7})$	M1		3.3
	[P(X, 9)] = 0.038292	A1		3.4
	[Significant result or reject H ₀]	A1		2.2b
	E.g. there is evidence to support Kamil's claim		(4)	
		(9 r	nark	s)
(a)	Notes B1 May use idea of independent events: a suitable reason, in context, cov			
	random packing or packets filled independently. Should mention key words/ideas of: prizes in packets or packets in boxes May use idea of constant probability. Must see key words underlined in scheme. Idea of probability with "independence" or "not affected by other packets" is B0 B0 for: Idea of only 2 cases. E.g. Packet contains a prize or not or Idea of a fixed number of trials. E.g. Need a fixed number of packets in each box			
(b)(i) (ii)	B1 for awrt 0.173 B1 for awrt 0.0616			
(c)	M1 for sight of B(5, "0.0616") or ${}^{5}C_{2}("0.0616")^{2}(1-"0.0616")^{3}$ ft their a	nswei	to (t	o)(ii).
	A1 for an answer in the range [0.0313 to 0.0314] Use of 0.0616 gives 0.031			
(d)	B1 for both hypotheses correct in terms of p or π M1 for selecting an appropriate model, may be implied by 1 st A1 or P($X = 9$) = 0.0199(2) 1 st A1 for 0.038 or better or allow 0.04 with sight of P(X , 9)			
ALT	Critical Region. Allow CR of X , 9 (or $X < 10$) provided a supporting pro			
	e.g. A1 for correct CR plus $P(X_1, 10) = 0.0718$ (accept 2sf or 1sf if probable 1.1)			
	2 nd A1 (dep on 1 st A1 but indep of hyp's) for a suitable conclusion in context that suggests support for (Kamil's) claim or states that there is evidence that proportion/probability/chance of packets containing a prize is less than ½			ests
	Do not award 2 nd A1 for contradictory statements e.g. "not significant" so "s	uppor	ts cla	nim"
Normal	Sight of N $\left(\frac{110}{7}, \frac{660}{49} \text{ or awrt } 13.5\right)$ or probability of 0.045(20) or 0.033			

Qu 3	Scheme	Marks	AO
(a)	Need to replace tr with a numerical value	M1	1.2
	Value of tr is between 0 and 0.05 suggest using e.g 0.025, 0 or value, 0.05	A1	1.1b
		(2)	
(b)(i)	$\left[\overline{x} = \frac{389.3 \sim 390.8}{184} \right] = 2.119 \text{awrt} \frac{2.12}{92} \text{allow} \frac{195}{92} \text{or } 2\frac{11}{92}$ $\left[\sigma = \right] \sqrt{\frac{(\text{awrt})4336}{184} - \overline{x}^2} \underline{\text{or}} \text{allow} \left[\sigma^2 = \right] \frac{(\text{awrt})4336}{184} - \overline{x}^2 \underline{\text{or}} \text{awrt} 19.1$	B1	1.1b
(ii)	$\left[\sigma = \right] \sqrt{\frac{(\text{awrt})4336}{184} - \ \overline{x}^2\ } \underline{\text{or}} \text{ allow } \left[\sigma^2 = \right] \frac{(\text{awrt})4336}{184} - \ \overline{x}^2\ \underline{\text{or}} \text{ awrt } 19.1$	M1	1.1b
	= 4.367 awrt 4.37	A1	1.1b
		(3)	
() ()		D.1	4 41
(c)(i)	Only covers May~Oct (so not a suitable sample)	B1	1.1b
(ii)	e.g. Winter months are <u>missing</u> when we'd expect <u>more rain</u> so expect estimate in (b)(i) to be an <u>underestimate</u> (oe)	B1	2.4
		(2)	
		(- 1	
	NT-4	(7 mark	<u>s)</u>
(a)	Notes M1 for recognising that tr must be replaced (oe) with a numerical value		
(a)	Wil for recognising that it must be replaced (be) with a numerical value		
	The following examples would score M0: The tr values are worth 0 so ignore or must remove outliers or fill gaps in table or make widths the same or need to A1 for using a suitable value: e.g. 0.025 (or allow 0) i.e. any value in [0, 0.05] (these give $\sum x = 390$ (3sf), use of 0.05 gives 390.8, use of 0 gives 389.3 at	o find mid- [5]	points
(b)(i)	or must remove outliers or fill gaps in table or make widths the same or need to A1 for using a suitable value: e.g. 0.025 (or allow 0) i.e. any value in [0, 0.05 (these give $\sum x = 390$ (3sf), use of 0.05 gives 390.8, use of 0 gives 389.3 at	o find mid- [5]	points
	or must remove outliers or fill gaps in table or make widths the same or need to A1 for using a suitable value: e.g. 0.025 (or allow 0) i.e. any value in [0, 0.05] (these give $\sum x = 390$ (3sf), use of 0.05 gives 390.8, use of 0 gives 389.3 at B1 for awrt 2.12 or allow simplified fraction or mixed number. B0 for $\frac{390}{184}$	o find mid- 5] llow in (b)	-points
(b)(i) (ii)	or must remove outliers or fill gaps in table or make widths the same or need to A1 for using a suitable value: e.g. 0.025 (or allow 0) i.e. any value in [0, 0.05 (these give $\sum x = 390$ (3sf), use of 0.05 gives 390.8, use of 0 gives 389.3 at	o find mid- 5] llow in (b)	-points
	or must remove outliers or fill gaps in table or make widths the same or need to A1 for using a suitable value: e.g. 0.025 (or allow 0) i.e. any value in [0, 0.05] (these give $\sum x = 390$ (3sf), use of 0.05 gives 390.8, use of 0 gives 389.3 at B1 for awrt 2.12 or allow simplified fraction or mixed number. B0 for $\frac{390}{184}$ M1 for a correct expression for standard deviation or variance. Allow $\sum x^2 = \frac{1}{184}$	o find mid- 5] llow in (b) awrt 4336	points

Part (c) can effectively be marked together.

- (c)(i) B1 for a comment mentioning that data is just from May~Oct (so not representative of the whole year).
 - Just saying "only 184 days so not representative" is B0, must mention May ~ Oct
 - (ii) B1 for comment that <u>missing/winter</u> months expected to have more rain (oe) **and** "underestimate"(oe)

We are looking for all 3 of these ideas here:

- 1. A statement or implication that missing data is from winter or different months.
- 2. A suggestion about the rainfall in these months (probably more rain).
- 3. A statement about the impact on the estimate in (b)(i) <u>equivalent</u> to saying it would be an underestimate or the (actual) mean will be higher.
- **SC** If you see "Leeming or N or NE has <u>less</u> rain in winter months" please send to review

Qu 4	Scheme	Marks	AO
(a)	[Let $N = \text{height from region } A$; $P(N > 180) =] 0.24937 awrt 0.249$	B1	1.1b
(b)	$H_0: \mu = 175.4$ $H_1: \mu \neq 175.4$	(1) B1	2.5
(0)	*	ы	2.5
	[S = height from region B] $\bar{S} \sim N\left(175.4, \frac{6.8^2}{52}\right)$ Allow $\sigma^2 = \text{awrt } 0.889$	M1	3.3
	$[P(\overline{S} > 177.2)] = 0.02814$	A1	3.4
	$[0.028 > 0.025$, Not sig, do not reject H_0	A1	2.2b
	<u>Insufficient</u> evidence to <u>support</u> student's <u>claim</u>	(4)	
		(-)	
(c)	$[p\text{-value} = 2 \times 0.02814 =] 0.05628$	B1ft	1.2
	in range $0.056 \sim 0.06$ or $5.6(\%) \sim 6(\%)$	(1)	
			,
	Notes	(6 mark	s)
(a)	B1 for awrt 0.249		
(b)	B1 for both hypotheses correct in terms of μ (See below for one-tail test)		
	M1 for selecting the correct model, may be implied by standardisation using may be implied by a correct <u>value</u> in 1 st A1	correct va	lues <u>or</u>
	e.g.(Prob =) 0.028 or awrt 0.972, (Z =) 1.9(08) (C	V=) 177.25	5
	Condone use of S (or any other letter) instead of \overline{S}		
	Condone use of $\overline{S} \sim N\left(177.2, \frac{6.8^2}{52}\right)$ but this will lose 2nd A mark		
	1^{st} A1 for probability of awrt 0.028 (allow 0.03 if P($\overline{S} > 177.2$) is seen)		
	Condone $1 - 0.02814 \dots = 0.9718 \dots$ (awrt 0972) only if clearly compa		0.975
ALT	Allow $Z = 1.9(088)$ and comparison with 1.96 (or better: calc gives 1.0 or CR of $\lceil \overline{S} \rceil$ 177.248(awrt 177.25) Allow $\lceil \overline{S} \rceil > 177.248$ (awrt 177.25)		
	Implied by diagram or correct interpretation of inequality with their C		
	(Ignore any attempt at a lower CR for \overline{S})	v	
	2^{nd} A1 (dep on 1st A1 and use of correct model. Use of N(177.2,) scores A0))	
	for a conclusion using context: e.g. does <u>not support</u> student's <u>claim</u>		
	or e.g. insufficient evidence of a difference in heights Do not allow 2 nd A mark for contradictory statements		
	e.g. "significant" so "no support for claim"		
(2)	P16t for angular in range 0.056, 0.06 at 5.60/ 60/ (Paraga are in the internal and interna	dono	m ~ 0/ \
(c)	for answer in range 0.056~0.06 or 5.6%~6% (Ranges are inclusive, con- (can ft their probability, provided < 0.5, from part (b) but not 0.025 leading		ng % <i>)</i>
NB	One-tail test [Max of 3/5 for (b) and (c)]	1 1/1 4 -	
	In (b) B0 (hypotheses) M1(model as above) 1^{st} A1[for probability or Z comparation of $\sum_{n=1}^{\infty} x_n > 176.95$ (awrt 177)] 2^{nd} A1 for conclusion in context that support		
	$CR\left[\overline{S}\right]$ or > 176.95 (awrt 177)] 2^{nd} A1 for conclusion in context that $\underline{\sup}$	ports cialli	1 OI.
	"heights of men from B is different from/greater than from A" In (c) B0		

Qu 5	Scheme	Marks	AO	
(a)	$P(S \cap \{X = 50\}) = P(S \cap \{X = 80\}) [= \text{a constant}, V] \Rightarrow b \times \frac{k}{50} = c \times \frac{k}{80}$ May see: $\frac{k}{50} = \frac{V}{b}$ and $\frac{k}{80} = \frac{V}{c}$ (condone any <u>letter</u> for V even S)	M1	3.1a	
	So $c = \frac{8}{5}b *$	A1cso*	1.1b	
(b)	$d = 2b$ or $a = \frac{2}{5}b$ or $c = 4a$ or $d = 5a$ or $d = \frac{5}{4}c$	M1 A1	2.1 3.3	
	$\frac{2}{5}b + b + \frac{8}{5}b + 2b = 1$	M1	2.1	
	$\Rightarrow 5b = 1$ so $b = \frac{1}{5}$ (o.e.)	A1	1.1b	
	$\underline{a = \frac{2}{25} b = \frac{1}{5} c = \frac{8}{25} d = \frac{2}{5}}$	A1 (5)	3.2a	
(c)	[Experiment suggests for Nav] $P(S \mid \{X = 100\}) = 0.3 \implies k = 30$	(3)		
	or $0.3 = \frac{V}{0.4} \Rightarrow V = 0.12$ So model won't work since	B1	2.4	
	$P(S \mid X = 20) = \frac{30}{20} \text{ or } \frac{0.12}{0.08} \text{ and so would be greater than 1}$	(1)	`	
	Notes	(8 marks	S)	
(a) *	M1 for use of $P(S X = x) \times P(X = x)$ for $x = 50$ and $x = 80$ (Must see Any expression or equation MUST be based on the probability st A1cso for rearranging to required result, no incorrect work seen, condon	tatements i	n qu.	
NB Use of values e.g. $b = \frac{50}{20 + 50 + 80 + 100}$ to prove (a) is M0A0 but scores 2 ⁿ			l in (b)	
(b)	Marks for (b) may be awarded for work seen in (a) 1st M1 for at least one other relationship (either probability the subject) from the list. 1st A1 for a second different relationship (either probability the subject) from the list. or Allow for: $\frac{ak}{20} = \frac{bk}{50} = \frac{ck}{80} = \frac{dk}{100}$ for 1st M1 1st A1 2nd M1 for using or stating sum of prob's = 1 May be implied by one correct probability. 2nd A1 for one correct probability e.g. $b = \frac{1}{5}$ or exact equivalent such as 0.2			
	3^{rd} A1 for all correct probabilities. Allow exact equivalents e.g. $c=0.32$ Sight of correct distribution or list of probs with no obvious incorrect	t probabilities. Allow exact equivalents e.g. $c = 0.32$ tribution or list of probs with no obvious incorrect working is 5/		
(c)	B1 for deducing $k = 30$ and giving a suitable example to show model by	breaks dov	vn	

Qu 6	Scheme	Marks	AO
(a)	$2 \times 4.2, 4 \times 4, 4 \times 3.5, 10 \times 1 (= 8.4 + 16 + 14 + 10 = 48.4)$	M1	1.1b
	[So P(10 < T < 30) =] $\left[\frac{48.4}{90}\right] = \frac{121}{225} = 0.53777$ 0.53~0.54 (2sf OK)	A1	1.1b
(b)	(Not suitable as) data is not symmetric <u>or</u> is skew (normal is symmetric) ("Even" distribution or a diagram <u>on its own</u> is not enough so B0)	(2) B1 (1)	2.4
(c)	$\int x e^{-x} \left(dx \right) = \int x d(-e^{-x})$	M1	2.1
	$= \left[-xe^{-x} \right] - \int \left(-e^{-x} \right) \left(dx \right) (+c)$	A1	1.1b
	$\int_{0}^{n} x e^{-x} (dx) = \left[-x e^{-x} - e^{-x} \right]_{0}^{n} = \left(-n e^{-n} - e^{-n} \right) - \left[-(0) - 1 \right]$	dM1	1.1b
	$= 1 - (n+1)e^{-n} $ (*)	A1cso* (4)	1.1b
(d)	Require area = 90 i.e. $k \int_{(0)}^{(n)} xe^{-x} dx = 90$ (ignore limits)	M1	3.1a
	Using the result in part (c) with $n = 4$ gives $k \lceil 1 - 5e^{-4} \rceil = 90$	M1	2.1
	(k=) 99 (.0729) (*)	A1cso*	1.1b
(e)(i)	[P(10 < T < 30) =] 0.64863 awrt <u>0.649</u>	B1 (1)	1.1b
(ii)	[No. of patients =] $(99) \left[(1-4e^{-3}) - (1-2e^{-1}) \right]$ (= 53.1)	M1	3.4
	Prob = $\frac{0.5366\times99}{90}$ = 0.59027[or 0.5907] = awrt 0.590 or 0.591	A1 (2)	3.2a
(f)	eg Patients might stay longer than 40 hours (Can ignore other comments unless clearly contradictory.)	B1 (1)	3.5b
	Notes	(14 mar	ks)
(a)	Notes M1 for an attempt to find the number between 10 and 30 (2 correct products	or 48 or 4	8.4 seen)
	A1 for 2sf answer in $[0.53 \sim 0.54]$ NB use of 48 gives 0.5333 [Correct and		
(b)	B1 for a comment suggesting not suitable based on (lack of) symmetry or '	'not bell sh	aped"
(c) *	1^{st} M1 for attempting integration by parts in right direction. Must have $u = x$ and $v = \pm e^{-x}$ 1^{st} A1 for a correct first step, correct first integration and expression for second integral 2^{nd} dM1 (dep on 1^{st} M1) for all integration attempted and some use of at least one limit 2^{nd} A1 for cso with no incorrect working seen. Minimum is correct int and use of limits seen.		
			SCOII.
(d) *	1 st M1 for realising need area under the curve (implied by the integral) = 90 2 nd M1 for use of (c) with $n = 4$ and set = 90 May be implied by sight of 99.07 or better A1cso for $k = 99$ or awrt 99.1		
NB	Allow use of $k = 99$ and show area = awrt 89.9 with a conclusion to	score 3/3	
(e)(i) (ii)	B1 for awrt 0.649 M1 for use of (c) with $n = 1$ and $n = 3$ Don't need the 99. Implied by sight of awrt 0.54 A1 for awrt 0.590 or awrt 0.591 Allow 0.59 from correct working seen.		
(f)	B1 eg for comment, in context, about the upper limit for $\underline{\text{time } (t \text{ or } x)}$ (time/ho	our may be	implied)

Notes on Question 5

The question essentially uses the definition of $P(A \mid B)$ given in the formula booklet.

In particular
$$P(S | \{X = x\}) = \frac{P(S \cap \{X = x\})}{P(X = x)}$$

The first "blob" tells us that $P(S | \{X = x\}) = \frac{k}{x}$ where k is a constant.

The second "blob" tells us that $P(S \cap \{X = x\})$ is the same for all x so $P(S \cap \{X = x\}) = V$ where V is a constant.

Using these results in
$$\boxed{1}$$
 gives $\frac{k}{x} = \frac{V}{P(X = x)}$ $\boxed{2}$

Line 1 of MS for part (a) uses
$$V = P(X = x) \times \frac{k}{x}$$
 for $x = 50$ and $x = 80$

Line 2 of MS for part (a) uses 2 with x = 50 and x = 80

Other implications

Equation 1 can be rearranged to give $P(X = x) = x \times \frac{V}{k}$ 3

So when
$$a+b+c+d=1$$
 is used this gives $1 = \frac{V}{k} (20+50+80+100)$ or $\frac{V}{k} = \frac{1}{250}$

In particular if we use this relationship in $\boxed{3}$ the probabilities a, b, c and d can simply be written down for example $b = \frac{50}{250}$ as given in the **NB** in the notes on the MS.

The point is that k and V will vary according to equation $\boxed{4}$ but as part (c) shows there are some restrictions on the values k, and therefore V, can take.

Since $\frac{k}{x}$ is a probability then, ignoring the trivial cases*, $0 < \frac{k}{x} < 1$ and the "restricting" value of x is clearly x = 20 so 0 < k < 20 and from 4 we get $0 < V < \frac{20}{250} = \frac{2}{25} = a$

So the restrictions on k and on V are given by the shortest distance and its associated probability.

* k = 0 would say Tisam can never get the ball in the cup no matter what the distance.

k = 20 says she always gets the ball in the cup for any distance.